Quality of Focus and Depth of Field in the HERA-B RICH Optical System

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Introduction

This note presents analytical methods for describing the focusing properties of the HERA-B RICH optical system, with emphasis on resolution in Čerenkov angle and depth of field. Our goal in pursuing this approach is to provide approximate formulas which can be used for understanding the focal plane design and interpreting results from monte carlo simulations.

The optical design employed in the HERA-B RICH—"parallel to point" focusing provided by spherical mirrors—is described in various HERA-B notes[1, 2] and the HERA-B proposal and technical design report[3]. The position on a *planar* detector, $\boldsymbol{d}$, of the intersection of a ray, reflected by a spherical mirror, that was initially heading in the direction, $\hat{\boldsymbol{u}}$, is given by:

$$\boldsymbol{d} = \frac{1}{(\hat{\boldsymbol{n}}_{\mathbf{0}} \cdot \hat{\boldsymbol{v}})} \hat{\boldsymbol{n}}_{\mathbf{0}} \times \left[\frac{1}{2}(1+2s)(\hat{\boldsymbol{u}} - \hat{\boldsymbol{u}}_{\mathbf{0}}) \times \hat{\boldsymbol{v}} + (\frac{b^2}{1+\sqrt{1-b^2}} - 2s\sqrt{1-b^2}) \hat{\boldsymbol{u}} \times \boldsymbol{b} \right]$$

with $\hat{\boldsymbol{v}}$ representing a unit vector parallel to the direction of the reflected ray:

$$\hat{\boldsymbol{v}} = (1-2b^2)\hat{\boldsymbol{u}} + 2\sqrt{1-b^2}\boldsymbol{b}$$

The initial ray direction and impact-parameter (perpendicular distance between the ray and the center-of-curvature of the spherical mirror), $\boldsymbol{b}$, are orthogonal, $\hat{\boldsymbol{u}} \cdot \boldsymbol{b} = 0$; the distance of closest approach of the ray to the center-of-curvature is $b = |\boldsymbol{b}|$. A unit-radius of curvature for the spherical mirror is assumed throughout. The detector is centered along the direction $\hat{\boldsymbol{u}}_{\mathbf{0}}$ from the center-of-curvature, a distance s outside the $R = 1/2$ sphere, which represents the paraxial focal surface of such a system; the unit vector $\hat{\boldsymbol{n}}_{\mathbf{0}}$ is normal to the plane of the detector surface. Figure 1 indicates the relevant quantities used in this note to describe RICH optics.

The job of the optical system is to bring Čerenkov photons radiated from a given track at the same azimuth, but at different locations in the radiator vessel, to a common focus at

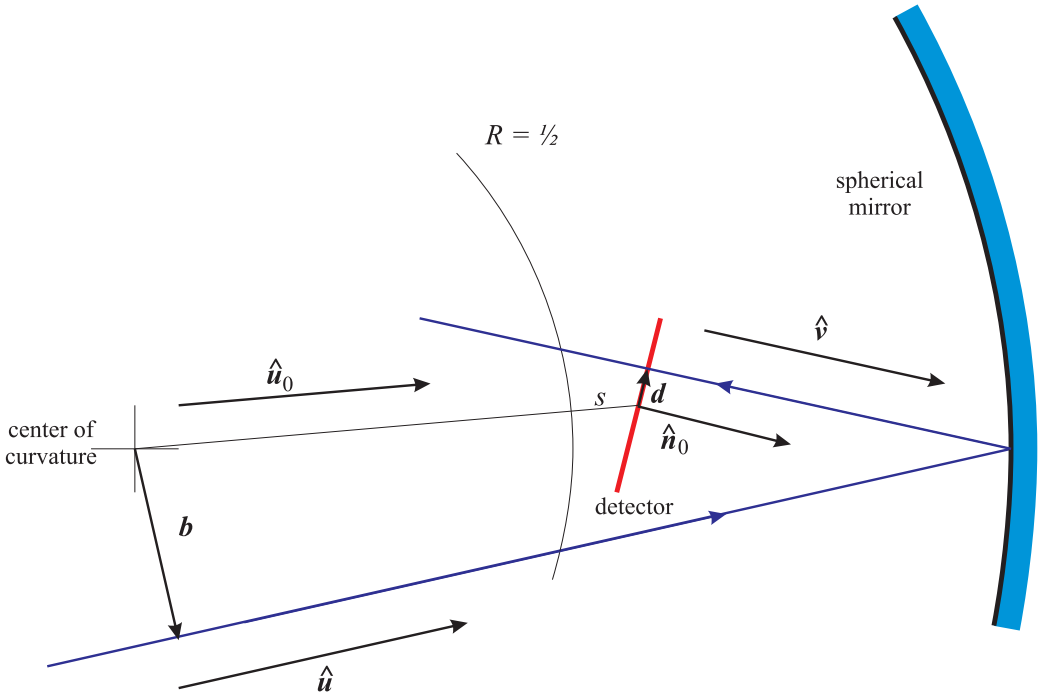

Figure 1: Schematic representation of HERA-B RICH optical system, indicating quantities described in this note.

the detector. Thus, at a particular azimuth (indicated by the unit vector $\hat{\boldsymbol{t}}$), the rays are nearly parallel to each other at the angle $\theta_{\check{C}}$ with respect to the original track direction. However, they have a spread of impact-parameters corresponding to extensions from the point where radiated to the point of closest approach to the center-of-curvature of the mirror. We describe the spread in impact parameters by a uniformly distributed random variable x ($-1 \leq x \leq 1$) and the vectors, $\boldsymbol{b_0}$,$\boldsymbol{\Delta}$, according to :

$$\boldsymbol{b} = \boldsymbol{b_0} + x\boldsymbol{\Delta}$$

In the case of HERA-B, assuming the parent track passes through the origin*, we have:

$$\boldsymbol{b_0} = -[\theta_t \hat{\boldsymbol{y}} + (1 - T/2)\theta_{\check{C}}\hat{\boldsymbol{t}}]$$

$$\boldsymbol{\Delta} = -\frac{T}{2}\theta_{\check{C}}\hat{\boldsymbol{t}}$$

where θ_t is the "tilt" angle—the angle between a line drawn through the center-of curvature of the mirror and the intersection of the proton beam direction with the mirror to the

*This is a good approximation for high-momentum tracks; the expression for $\boldsymbol{b_0}$ can be modified appropriately for tracks of moderate momenta, where bending by the dipole magnet displaces the apparent origin.

beam direction—of the spherical mirror, T is the thickness of the radiator vessel (in units of the mirror radius-of-curvature) and $\hat{\boldsymbol{y}}$ points in the vertical direction—the direction of mirror tilt. Strictly speaking, the projections of $\boldsymbol{b_0}$ and $\boldsymbol{\Delta}$ perpendicular to $\hat{\boldsymbol{u}}$ should be used, but, because of the small angles involved, the above expressions provide adequate approximations.

Dispersion in the radiator gas results in a spread of Čerenkov angles for emitted photons. An analysis of dispersion effects[4] indicates that the distribution of Čerenkov angles in the HERA-B RICH system will be approximately gaussian about the mean Čerenkov angle, $\theta_{\check{C}}$, with an RMS spread, $\sigma_{\check{C}} \approx \theta_{\check{C}}/100$.

Focusing in the HERA-B RICH as a beam transport problem

If we consider the set of rays to be focused as a "beam" centered around the direction $\hat{\boldsymbol{u}}$ passing a distance $\boldsymbol{b_0}$ from the center of curvature, then a given ray will have a direction error $\delta\hat{\boldsymbol{u}} = \delta\theta_{\check{C}}\hat{\boldsymbol{t}}$ and an (uncorrelated) impact parameter error $\delta\boldsymbol{b} = x\boldsymbol{\Delta} = -x\frac{T}{2}\theta_{\check{C}}\hat{\boldsymbol{t}}$ with respect to the beam average. For a bundle of rays associated with the "beam" direction $\hat{\boldsymbol{u}}_{\boldsymbol{0}}$, the detector coordinate for a ray with errors $(\delta\hat{\boldsymbol{u}}, \delta\boldsymbol{b})$ will be

$$\boldsymbol{d} = \frac{1}{2}(1+2s)\delta\hat{\boldsymbol{u}} + (2s - b^2/2)(\boldsymbol{b_0} + \delta\boldsymbol{b})$$

where we have kept the lowest, non-vanishing terms describing spherical aberrations and assumed the detector plane is oriented normal to the average rays striking it in the vicinity of $\hat{\boldsymbol{u}}_{\boldsymbol{0}}$. This last requirement is equivalent to setting

$$\hat{\boldsymbol{n}}_{\boldsymbol{0}} = (1 - 2b_0^2)\hat{\boldsymbol{u}}_{\boldsymbol{0}} + 2\sqrt{1 - b_0^2}\boldsymbol{b_0}$$

The best focus for these rays is achieved by positioning the detector at a distance s_0 from the $R = 1/2$ sphere such that the RMS spot size, $\sigma_d = (\langle d^2\rangle - |\langle \boldsymbol{d}\rangle|^2)^{1/2}$ is minimized. The angle brackets, $\langle -\rangle$, indicate an ensemble average of the quantity enclosed.

It is seen that imaging Čerenkov photons in the HERA-B RICH reduces to a 1-dimensional beam-transport problem involving an ensemble of rays lying in the $\hat{\boldsymbol{u}}_{\boldsymbol{0}} - \hat{\boldsymbol{t}}$ plane, with a (nearly) invariant phase-space area described by $(\delta\hat{\boldsymbol{u}}, \delta\boldsymbol{b})$. In analogy to beam-transport optics in accelerators, we choose to describe the focusing properties of the RICH with a characteristic "β-function" that depends on detector location s and a beam "emittance" ϵ that is a measure of the phase-space density of the bundle of rays. The effects of the spherical mirror and drift spaces in transporting photons from their point of origin in the radiator vessel to a focus on the detector plane is to "rotate" the beam phase-space by approximately $\pi/2$. Spherical aberrations and optical imperfections in the mirrors have the effect of diluting the phase-space density and distorting the β-function in the vicinity of the detector. The RMS spot size at the detector is related to the beam emittance and β-function by

$$\sigma_d^2 = \epsilon\beta(s) = \epsilon\left[\beta^* + \frac{(s - s_0)^2}{\beta^*}\right]$$

where the explicit form of the β-function in a drift-section with a waist at the location s_0 is used. The constant β^* is the value of the β-function at the focus; it is a measure of the depth-of-field of the optical system because—true for any beam-transport system involving only a drift-section with no focusing elements—the β-function will increase by a factor of $\sqrt{2}$ when moving by a distance $\pm\beta^*$ from s_0, the best focus. The RMS spread in angles of rays at the focus—and throughout the drift region—is given by $\sigma_\theta = \sqrt{\epsilon/\beta^*}$.

In the paraxial approximation, where spherical aberrations and imperfections in the mirrors are ignored, elementary considerations give:

$$s_0 = 0$$

$$\beta^* = \frac{\sqrt{3}\sigma_{\check{C}}}{4\Delta}$$

$$\epsilon = \frac{\sigma_{\check{C}}\Delta}{\sqrt{3}}$$

where $\Delta = |\mathbf{\Delta}| = \frac{T\theta_{\check{C}}}{2}$. The factor $\sqrt{3}$ enters these expressions because $\langle x^2 \rangle = \frac{1}{3}$. With these optical parameters, we recover the usual performance characteristics for paraxial rays, $\sigma_d = \frac{1}{2}(\sigma_{\check{C}}^2 + 4s^2T^2\theta_{\check{C}}^2/3)^{1/2}$ and $\sigma_\theta = T\theta_{\check{C}}/\sqrt{3}$.

When spherical aberrations are included in the analysis[†] the corresponding expressions for the optical properties are:

$$s_0 = \frac{1}{4}\left[\frac{\langle b^4\rangle - \langle \boldsymbol{b}\rangle \cdot \langle b^2\boldsymbol{b}\rangle - \sigma_{\check{C}}^2/2}{\langle b^2\rangle - |\langle \boldsymbol{b}\rangle|^2 + \sigma_{\check{C}}^2/4}\right]$$

$$\beta^* = \left[\frac{\sigma_{\check{C}}^2 + \langle b^6\rangle - |\langle b^2\boldsymbol{b}\rangle|^2}{16(\langle b^2\rangle - |\langle \boldsymbol{b}\rangle|^2 + \sigma_{\check{C}}^2/4)} - s_0^2\right]^{1/2}$$

$$\epsilon = \left[(\sigma_{\check{C}}^2 + \langle b^6\rangle - |\langle b^2\boldsymbol{b}\rangle|^2)(\langle b^2\rangle - |\langle \boldsymbol{b}\rangle|^2 + \sigma_{\check{C}}^2/4) - (\langle b^4\rangle - \langle \boldsymbol{b}\rangle \cdot \langle b^2\boldsymbol{b}\rangle - \sigma_{\check{C}}^2/2)^2\right]^{1/2}$$

The various ensemble averages to be carried out in the above expressions are straightforward, albeit tedious, and the resulting formulas are somewhat complicated. They can be simplified by keeping only the most significant terms, which, in the case of the HERA-B RICH, are found by noting $b_0 > \Delta > \sigma_{\check{C}}$. With these simplifications, the RICH focusing system can be described by the following characteristics:

$$s_0 \approx \frac{1}{2}\left[(\boldsymbol{b_0}\cdot\hat{\boldsymbol{t}})^2 + \frac{b_0^2}{2} - \frac{3}{4}\left(\frac{\sigma_{\check{C}}}{\Delta}\right)^2\right]$$

$$\beta^* \approx \frac{\sqrt{3}\sigma_{\check{C}}}{4\Delta}\left\{1 + \frac{4}{3}\left(\frac{\Delta}{\sigma_{\check{C}}}\right)^2 (\boldsymbol{b_0}\cdot\hat{\boldsymbol{t}})^2\left[b_0^2 - (\boldsymbol{b_0}\cdot\hat{\boldsymbol{t}})^2\right]\right\}^{1/2}$$

[†]Note: optical imperfections in the mirrors should simply be added in quadrature to the RMS Čerenkov angle; they effect the focusing only through the parameter $\sigma_{\check{C}}$.

$$\epsilon \approx \frac{\sigma_{\check{C}}\Delta}{\sqrt{3}} \left\{ 1 + \frac{4}{3}\left(\frac{\Delta}{\sigma_{\check{C}}}\right)^2 (\boldsymbol{b_0}\cdot\hat{\boldsymbol{t}})^2 \left[b_0^2 - (\boldsymbol{b_0}\cdot\hat{\boldsymbol{t}})^2 \right] \right\}^{1/2}$$

Spherical aberrations affect β^* and the emittance, ϵ, by the same factor. This result is obvious because the effect of spherical aberrations is to increase the spot size, while the angular divergence of the reflected beam is not changed from the linear-optics case. The maximum change to β^* or ϵ is $\left[1 + \frac{1}{3}\left(\frac{b_0^2\Delta}{\sigma_{\check{C}}}\right)^2\right]^{1/2}$, which is only a few-% effect for typical HERA-B RICH parameters. This result depends, however, on the details of the optical design and with different parameters, β^* and ϵ could be more sensitive to spherical aberrations.

Numerical results

Values of the characteristic RICH optical constants for typical HERA-B design parameters ($R = 11.4$ m, $\theta_{\rm tilt} = 9$ degrees, $T = 1/4$) are given as a function of Čerenkov azimuth in Figure 2. It is seen that the optimum detector position, s_0 is quite sensitive to spherical aberrations—through the azimuth-dependence of this parameter—while β^* and ϵ are insensitive, as discussed previously. Indeed, for HERA-B parameters, the variation in s_0 for different photon azimuths is comparable to β^*, meaning that there will be some degradation in resolution due to the fact that the detector position is fixed, while the optimum focus will "move around" according to azimuth of the radiated photons.

The dependence of the resolution (expressed as the resolution in Čerenkov angle, averaged over the azimuth of emitted photons) on detector location is shown in Figure 3. As expected, the average resolution is seen to degrade by roughly 10% when the detector position is varied by $\pm\beta^*/2 = \pm 20$ cm from the position with the best focus. At the best average focus, spherical aberrations increase the resolution from about 0.51 mr to 0.56 mr, assuming a uniform distribution in azimuth. For reference, the corresponding resolution in width of the Čerenkov ring due to the finite detector granularity is 0.45 mr for 9 mm square detector cells (at the field lens of the light collectors); optical errors introduced by imperfections in the spherical mirrors are negligible in comparison.

The actual detector location suggested by this calculation—$18 - 25$ cm outside the $R = 1/2$ sphere—is very close to that proposed by Marco Starič[5] from monte carlo calculations using typical photon distributions expected in the HERA-B RICH, but neglecting dispersion.

Conclusions

All of the results presented here were known from previous analytical and monte carlo studies, but the identification of beam-transport parameters for the RICH optical system gives a simple, intuitive picture of its performance, particularly the depth-of-field. The

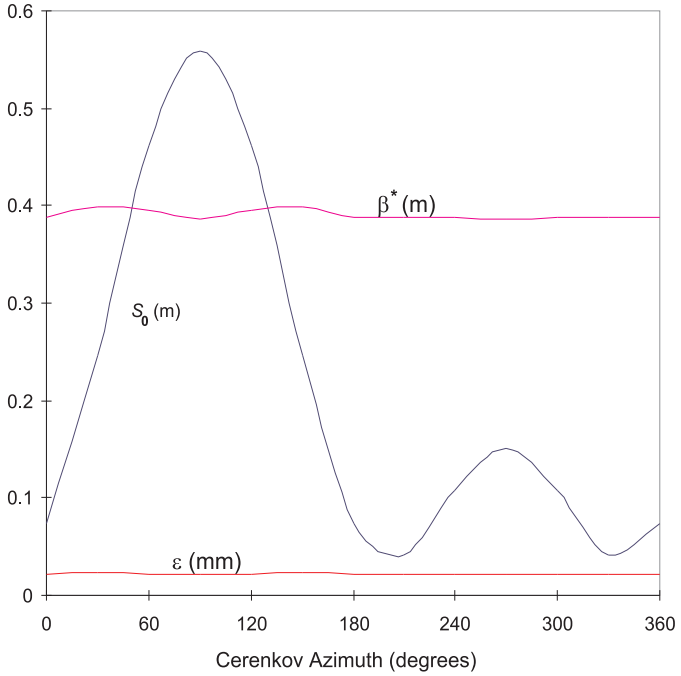

Figure 2: HERA-B RICH optical parameters as a function of Čerenkov azimuth for $R = 11.4$ m, $\theta_{\rm tilt} = 9$ degrees, $T = 1/4$.

interplay of dispersion, radiator vessel thickness, and tilt-angle gives an effective $\beta^* = 40$cm, implying that any degradation in ultimate performance will be less than 1 % as long as the detector position is within 6 cm of the optimum focus. By the same argument, the tolerance on radius-of-curvature of the spherical mirrors can be as large as ± 12 cm[‡].

[‡]The mirrors are aligned to a common spherical surface to within millimeters; the spread in the location of their respective $R = 1/2$ spheres will be 1/2 their spread in radii.

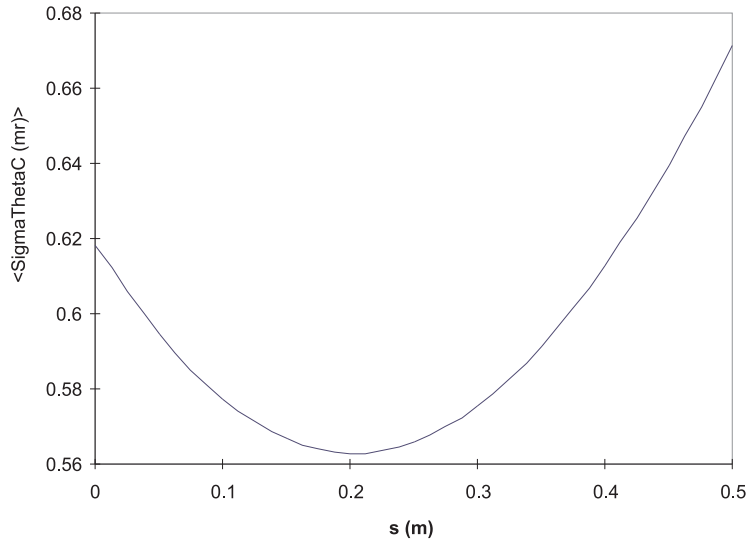

Figure 3: RMS resolution in Čerenkov angle, averaged over photon azimuths, $\langle \sigma_{\check{C}} \rangle$, vs detector position.

References

[1] Peter Križan and Marco Starič, *Optical Error of the HERA-B RICH*, Hera-B Note **95-131 RICH**.

[2] R. Schwitters, *HERA-B RICH Optics*, Hera-B Note **96-265 RICH**.

[3] *HERA-B Technical Design Report*, **DESY-PRC 95/01**, January 1995.

[4] R. Schwitters, *Comments on Chromatic Effects in the HERA-B RICH*, January 1997.

[5] Marco Starič, R5700 position for photon detector, April 1997.