Luminosity error calculations

Let's consider formula for luminosity L:

$$(1) \qquad \mathbf{L} = \frac{\mathbf{N}}{\boldsymbol{\sigma}} \cdot \frac{\boldsymbol{\lambda}}{(\mathbf{1} - \mathbf{e}^{-\lambda})},$$

where **N** - number of triggered events, **σ** – inelastic cross section and average number of interaction per one bunch crossing **λ** is determined:

$$(1) \qquad \boldsymbol{\lambda} = \frac{\mathbf{K} * \mathbf{R}}{\mathbf{8.523}}.$$

Here: **K**- the hodoscopes correction factor, **R**- hodoscopes rate.
Therefore we can say that luminosity is a function of these variables, i.e.

$$(3) \qquad \mathbf{L=L(N, K, R, \sigma)\ ;}$$

If each of these variables is taking with some error:

$$\mathbf{N = N \pm \Delta N,}$$
$$\mathbf{K = K \pm \Delta K,}$$
$$\mathbf{R = R \pm \Delta R,}$$
$$(4) \qquad \boldsymbol{\sigma = \sigma \pm \Delta \sigma;}$$

then the experimental errors (both statistical and systematic) **ΔL** for the luminosity should be calculated by formula:

$$(5) \qquad \Delta \mathbf{L} = \sqrt{\Delta \mathbf{L}_{\mathbf{N}}^{2} + \Delta \mathbf{L}_{\mathbf{K}}^{2} + \Delta \mathbf{L}_{\mathbf{R}}^{2} + \Delta \mathbf{L}_{\boldsymbol{\sigma}}^{2}},$$

where (taking into account (1)):

$$(6^{\mathrm{I}}) \qquad \Delta \mathbf{L}_{\mathbf{N}} = \frac{\mathbf{d}}{\mathbf{dN}} \mathbf{L(N,K,R,\sigma)} \cdot \Delta \mathbf{N} = \mathbf{L} \cdot \frac{\Delta \mathbf{N}}{\mathbf{N}};$$

$$(6^{\mathrm{II}}) \qquad \Delta \mathbf{L}_{\mathbf{K}} = \frac{\mathbf{d}}{\mathbf{dK}} \mathbf{L(N,K,R,\sigma)} \cdot \Delta \mathbf{K} = \mathbf{L} \cdot \frac{\mathbf{1} - \mathbf{e}^{-\lambda} - \boldsymbol{\lambda} \cdot \mathbf{e}^{-\lambda}}{\mathbf{1} - \mathbf{e}^{-\lambda}} \frac{\Delta \mathbf{K}}{\mathbf{K}};$$

$$(6^{\mathrm{III}}) \qquad \Delta \mathbf{L}_{\mathbf{R}} = \frac{\mathbf{d}}{\mathbf{dR}} \mathbf{L(N,K,R,\sigma)} \cdot \Delta \mathbf{R} = \mathbf{L} \cdot \frac{\mathbf{1} - \mathbf{e}^{-\lambda} - \boldsymbol{\lambda} \cdot \mathbf{e}^{-\lambda}}{\mathbf{1} - \mathbf{e}^{-\lambda}} \frac{\Delta \mathbf{R}}{\mathbf{R}};$$

$$(6^{\mathrm{IV}}) \qquad \Delta \mathbf{L}_{\boldsymbol{\sigma}} = \frac{\mathbf{d}}{\mathbf{d\sigma}} \mathbf{L(N,K,R,\sigma)} \cdot \Delta \boldsymbol{\sigma} = \mathbf{L} \cdot \frac{\Delta \boldsymbol{\sigma}}{\boldsymbol{\sigma}};$$

For example, for Ti target:

Statistic errors:

ΔN = √**N;**
Δσ = Ti cross section (i.e., σ=645±9.0);
ΔR = $t^{-1} \cdot \sqrt{0.012} * \Sigma$(Interaction rates*1sec), where 0.012 is the hodoscopes acceptance and t is time (in sec);
ΔK = hodoscopes correction factor **K**=1.412±0.034 (determined by S.Somov);

Systematic errors:

$\Delta N = N_{tot} - N_{rec}$; difference between taped and reconstructed events;
$\Delta\sigma$ – from the published cross section values;
ΔR – from interaction rate measured by Hodoscopes, Foil-detectors, Luminosity-monitors and Charge Integrators connected to the target.
ΔK – four different methods used to determine the hodoscopes correction factor (energy deposition, hit distributions etc)

11.10.2003. aushev@mail.desy.de